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BOOK REVIEW

Pattern Formation: An Introduction to Methods. By REBECCA HOYLE. Cambridge University Press, 2006. 422 pp. ISBN 0 521 81750 1. £45.

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This book provides a valuable introduction to, and survey of, mathematical methods used to describe the formation and dynamics of natural patterns. The author has succeeded in writing an applied mathematics textbook, aimed at final year undergraduates and graduate students, which also contains enough detail to engage a broad cross-section of researchers in applied science.

'Pattern formation' is taken here to mean the emergence of spatial structure in dissipative, externally forced, continuum systems. The book develops 'model independent' methods that rely on low-dimensional bifurcation theory, and which provide a bridge between finite-dimensional dynamics and spatio-temporally complicated behaviour. Model independence allows the author to bring together traditional examples of instability from fluid mechanics (Rayleigh–Bénard convection, Faraday waves, the Benjamin–Feir instability) with other well-studied experimental phenomena including chemical spiral waves, quasi-patterns, and the various kinds of pattern defect that arise in large domains.

In keeping with the photogenic qualities of the topic, Chapter 1 contains a selection of well-reproduced colour illustrations, and motivates the subject with brief general overviews of Rayleigh–Bénard convection, reaction–diffusion systems, including the Belousov–Zhabotinsky reaction and Turing instability, and (very briefly) Faraday waves at the free surface of a vertically vibrated fluid layer. Chapter 2 presents a correspondingly brief, and informal, summary of basic elements of undergraduate bifurcation theory. Chapters 3–6 deal with exactly space-periodic patterns and develop the theory of bifurcations in the presence of symmetry, as applied to patterns that are periodic in one or two space dimensions. The language of groups and representations is developed in an accessible and deliberately informal style and applied to a series of symmetric bifurcation problems of increasing complexity. This is probably the part of the book with which a generally interested applied mathematician would be least familiar.

Chapters 7–10 deal with patterns that, while locally regular, depart from spatial periodicity over long lengthscales. Taking the small parameter to be the pattern amplitude, a systematic multiple-scales approach enables treatment of the various modulational instabilities, beginning with the Eckhaus and zig-zag instabilities of stripe patterns and ending with complicated, and still not completely resolved, cases such as the influence of large-scale mean flows and conservation laws. A final chapter is devoted to the 'Cross–Newell' formalism for fully nonlinear but spatially slowly varying patterns, in which the reciprocal of the aspect ratio is used as the small expansion parameter. There is also discussion of the more abrupt changes in pattern orientation and amplitude which occur at grain boundaries and at defects.

This book achieves a great deal in its attempt to provide a student textbook, both by its style and by the provision of end-of-chapter exercises. The majority of the analysis is developed with the Rayleigh–Bénard problem in mind, and this serves the author and reader well since the aspects of this canonical system that are worthy of discussion are so many and varied. It is an indication of the breadth of the subject, rather than the narrowness of the remit of the book, that several notable pattern-forming systems are scarcely mentioned, for example Taylor–Couette flow, nonlinear optics, liquid-crystal electroconvection, and double-diffusive convection. Faraday waves and reaction–diffusion systems appear in occasional enlightening cameos. It is a pity, given the back cover blurb, that neither sand ripples nor zebra stripes is discussed. Also omitted are localised patterns and the various kinds of spatiotemporal chaos that occur, for example, in the complex Ginzburg–Landau equation.

The bibliography contains approximately two hundred references and provides a very good guide to the recent research literature; a few standard references, for example the excellent two volumes by Murray (2001), do not appear. In summary, this book provides an excellent first look at the traditional applications and the mathematical theory of pattern formation. Readers who work through a substantial part of the book will be well equipped to dive into the current research literature.

REFERENCE

Murray, J. D. 2001 *Mathematical Biology*, Vols. I and II, 3rd Edn. Series in Interdisciplinary Applied Mathematics, Vols. 17 and 18. Springer.

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